Statistical Characteristics of Enjoyable Race Games

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1. Introduction

Single-track race games played with dice form a wide class. They range from the simplest games, played on a track where the playing spaces are all the same, through games subject to simple instructions to go to another space, such as *Snakes and Ladders* through to the many variants of the *Jeu de l'Oie (Game of Goose, Gioco dell'Oca)*, where more complicated rules are added. In this class of game, known in the USA as 'roll-and-move', the moves of each player's token are strictly determined by the throw of one or more dice, so that there is no player choice. This makes it possible to 'play' such games completely by computer simulation, using the built-in random number generation facilities instead of dice - the well-known 'Monte Carlo' model. Another possibility - again using computers - is to specify completely the probability after any given number of throws that a particular space will be occupied, simply by chaining together the probabilities generated by each successive throw: the 'Markov chain' model. In this paper, these techniques will be used to address some aesthetic questions about race games:

- what makes a game 'playable'?

- what makes a game 'interesting'?

- do 'successful' games share common features?

No originality is claimed for the mathematical techniques used and the paper concentrates on the results obtained.

2. Markov Probabilities for *Snakes and Ladders*

As an illustration of the Markov approach, consider the game of *Snakes and Ladders,* in the form registered by F.H.Ayres of London in 1892 [Love, 1978]. There are 100 spaces, here arranged in a spiral track, though the track shape is unimportant. The five ladders go from 6 to 40, 20 to 90, 23 to 54, 44 to 70, and 60 to 95; and the five snakes go from 97 to 10, 83 to 33, 76 to 21, 58 to 27, and 48 to 15. The object is to reach space 100 exactly - overthrows mean that the player does not move. Ignoring for the moment the rule that throwing a six gives a free throw, the Markov chain probabilities for a single player are shown in figure 1 for 1, 2, 3, 5, 10 and 20 throws. The effect of the snakes and the ladders in feeding the token into different points on the track is clearly visible. Also clear is the fact that after a sufficient number of throws the probability graph settles down into a characteristic shape.

The model easily gives the probability of reaching square 100 after any given number of throws. This probability can be plotted against the number of throws to give the statistical distribution of the number of throws to finish the game. Figure 2 shows this distribution, both with and without the free throw after throwing a six. It is interesting that the distributions are not much different, despite an apparently significant rule variation.

3. Monte-Carlo results for *Snakes and Ladders*

Similar results for number of throws to finish can be obtained using the Monte-Carlo method, as shown in figure 3, which is based on the observed number of throws required to finish in 1000 games. The graph is not as smooth as in the Markov model, which gives exact probabilities, because the Monte-Carlo method produces results as in a real set of games and is therefore subject to statistical fluctuation.

Figure 4 shows a different set of observations for 1000 games using the Monte-Carlo method, namely the frequency distribution of where the token was observed after each throw. This is equivalent to taking a snapshot of the board after each throw and superimposing the results. (A similar result could be obtained by superimposing the Markov graphs for all successive throws). This kind of graph shows clearly how the 'snakes' and the 'ladders' have broken up the uniformity of the track. It also emphasises the crowding of spaces at the end of the track, caused by the exact-finish requirement.

4. Multiple Players

The above illustrations have been for a single 'player'. When the game is played by more than one, the Markov and Monte-Carlo techniques still apply. Interference between players has to be considered, of course. However, where - as in the version of *Snakes and Ladders* under consideration - players simply change places when there is a hit, no complications arise: the winner is different but the statistics are unaltered. The main effect of having multiple players is that the end-of-board pile up is reduced. However, as will be seen below, in other games interference can be more complicated.

5. Building *Snakes and Ladders* from Simpler Games

It is interesting to see how a moderately complicated game like *Snakes and Ladders* can be developed from a simple race game by adding rules. The aim of this is to show the effect of the rules on the distributions introduced above: there is no suggestion that the game was historically developed in the way indicated. Figure 5 shows, using the Monte-Carlo method, such development of a game with four players, using a single die:

> (a) beginning with an undifferentiated track of 100 spaces, without an exact finish - a game labelled in this paper as '*Long Serpent*'
>
> (b) adding the requirement of an exact finish - a game labelled as *'Exact Long Serpent'*
>
> (c) introducing the snakes and ladders – the *'Snakes and Ladders (Ayres)'* game.

In each case, the upper diagram shows the frequency distribution of positions on the board - the ''superimposed snapshots' of the board; the lower diagram shows the number of rounds (throws per player) required to win.

It is striking how the rule changes have changed the distribution on the board from that of the simple track. Also very striking is how the changes have broadened the distribution of number of rounds to finish.

6. Building the *Game of the Goose* from Simpler Games

Figure 6 shows a similar approach being used to build the classical *Game of the Goose* [Seville 1999]. Here, the game is of 63 squares and double dice are used.

> (a) begins with an undifferentiated track, without an exact finish - a game labelled here as *'Serpent';*
>
> (b) adds the requirement of an exact finish, with the rule that overthrows are counted backwards - a game labelled here as *'Reverse Overthrow Serpent';*
>
> (c) shows the complete game, with the favourable Goose squares and the various classical hazards - *'Goose'.*

As before, the left hand column shows the frequency distribution of positions on the board; the right hand column shows the number of rounds (throws per player) required to win.

Again, the distribution on the board has been strikingly changed by the rules. And again the distribution of number of rounds to finish has been very strikingly broadened.

A particular feature of the *Game of the Goose* is the effect of the traps at 31 (the well) and 52 (the prison), where a player must wait until released by another landing on the square. These give pronounced peaks in the distribution on the board. The other usual hazards are a *bridge* – go on to 12; 19, an *inn* – lose two turns; 42, a *maze* – go back to 39 (usually); *death* – start again. The image of a *goose* is found on the spaces of two interleaved sequences: 5,14,23 … and 9. 18, 27 … On landing on a goose space, the player doubles the throw, in the current direction of travel (which may be backwards if counting overthrows near the end).

7. Comparison of *Snakes and Ladders* and the *Game of the Goose* with simple race games

It is interesting to compare Figures 5(c) and 6(c). There are distinct similarities in the distributions both on the board and for the number of rounds. It is suggested that these similarities are not accidental. Both *Snakes and Ladders* and *Goose* achieve excitement and interest:

> - by introducing rule-based differentiation within the track itself, so that not all the playing spaces are the same
>
> - by introducing an exact finish requirement, though this is different in the two games

- by using rules that allow on occasion a very quick result and (more rarely) allow long-drawn-out games.

Without such features, a race game is so easy to model that even a casual observer watching the game, without using statistics or a computer model, can “see what is happening”: the group of tokens moves on average at a steady rate down the track, gradually spreading out as it does so. The winner may indeed be any one of the group, so the game is unpredictable to that extent, but there is no real excitement and the game soon palls.

By contrast, in both *Snakes and Ladders* and *Goose,* there is sufficient complexity for the game to seem unpredictable to the observer: the underlying statistical regularities are not obvious to the casual eye. Both games appeal to the human sense of hubris, in that each provides a mechanism whereby a player leading the field can be sent back (respectively, a long snake near the end; and the death space).

8. Statistical Comparisons

The comparisons are however more than just qualitative. Quantitative similarities can be demonstrated by statistical analysis, as summarised in Figure 7. This shows the average number of rounds to finish for all the games discussed, together with the standard deviation of the distribution, giving a measure of its spread.

Thus, for four players, the average number of rounds to win is about 17 for *Snakes and Ladders* and about 15 for *Game of the Goose* - whereas the underlying simple games are respectively much longer (29 rounds) and much shorter (only 9 rounds). Furthermore, the standard deviation of the average number of rounds to win is about 8 for *Snakes and Ladders* and about 11 for *Game of the Goose* - both very much larger than for the underlying simple games.

The convergence of these quantities for 4-player *Game of the Goose* and 4-player *Snakes and Ladders* is remarkable, given that these two games have totally different historical pedigrees. Presumably the characteristics of the games were developed by trial and error, with human enjoyment as the selection parameter.

It is also noteworthy that both games play well for numbers other than four. However, the statistical characteristics of the *Game of the Goose* are much affected by the existence of the traps (the well and the prison). These significantly increase the length of the game for three players, to an average of about 31 rounds - and for two players there is the possibility of a draw if both fall into different traps. For this reason, statistics for two-player *Game of the Goose* have not been presented here.

9. Conclusion

The perhaps foreseeable conclusion is that these two games were developed with similar playing considerations in mind: to produce excitement and variety, while being of a convenient length for practical play. Such a conclusion is nevertheless of some interest, since the *Game of the Goose* - of Italian origin - has in all likelihood a cabalistic significance as a 'game of life', while *Snakes and Ladders* began in Asia as a game of morals.

Further work is needed to determine how far the common statistical characteristics are shared by other successful race games - and whether now-forgotten games had undesirable playing characteristics that contributed to their downfall. Some work has been undertaken on *The Mansion of Happiness,* a game adapted from a British original but of great importance in the history of board games in the United States of America. [Whitehill 1999] As a game for four players, this is significantly slower than those mentioned above, taking about 25 rounds to win, though the distribution of numbers of rounds to win is of the same general shape as in those games.

References

Love, 1978

Brian Love, 'Play the Game', Michael Joseph, London 1978
The Ayres game is reproduced at pages 22-23.

Seville 1999

Adrian Seville, 'Tradition and Variation in the Game of Goose'
Board Games in Academia III, colloquium in Florence April 1999

Whitehill 1999

Bruce Whitehill, 'American Games: a Historical Perspective'
Board Games in Academia III, colloquium in Florence April 1999

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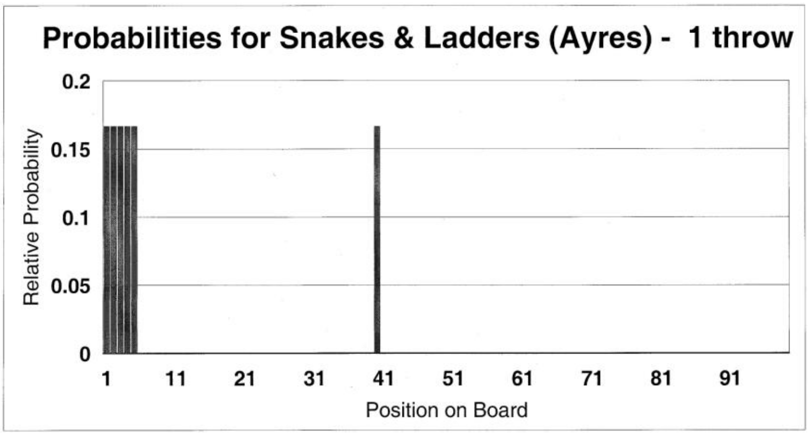
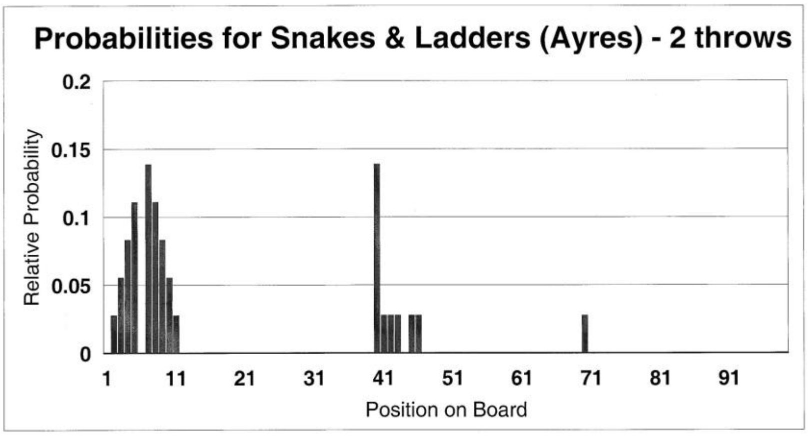
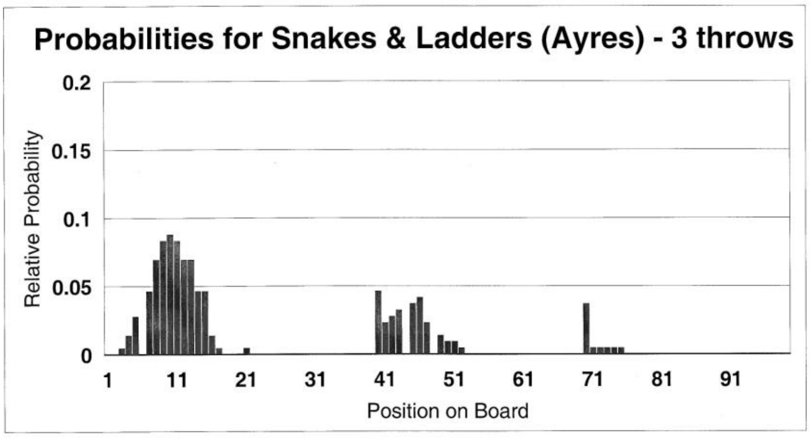
Figure 1 – Markov Probabilities for Snakes and Ladders







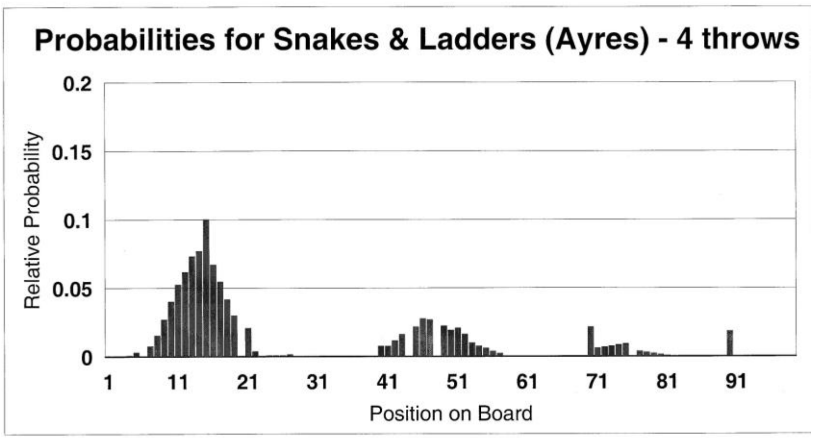
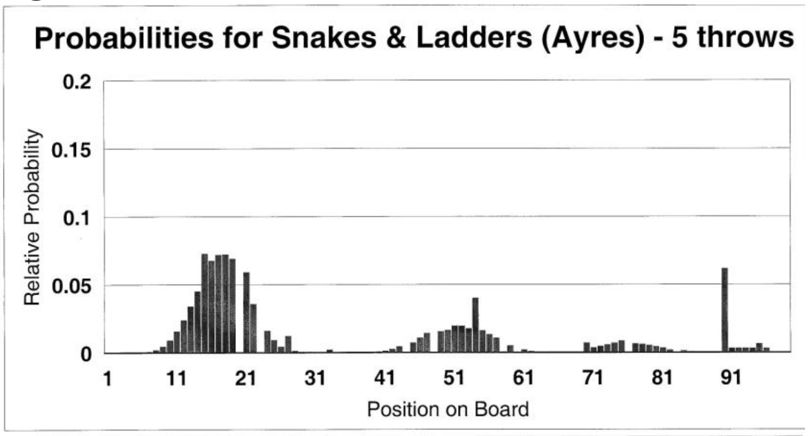
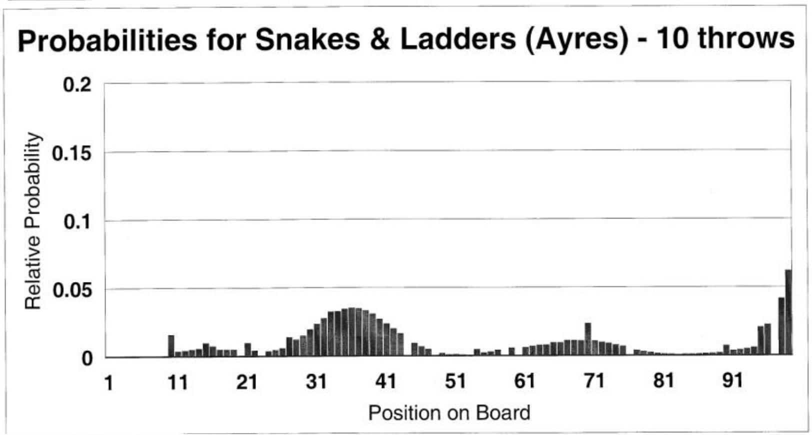
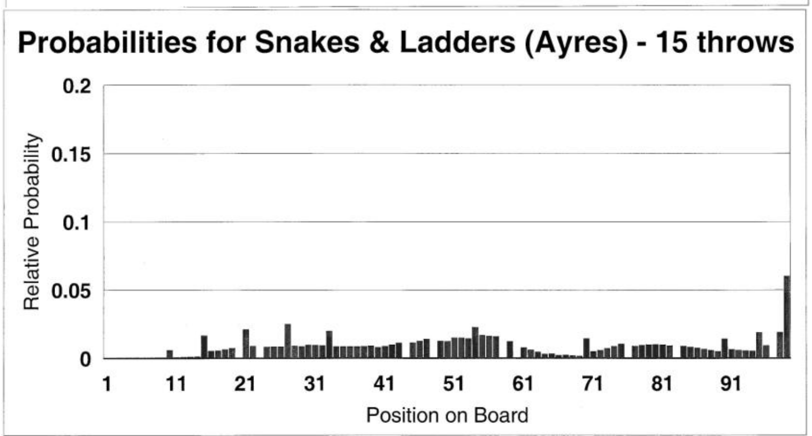


Figure 1 continued







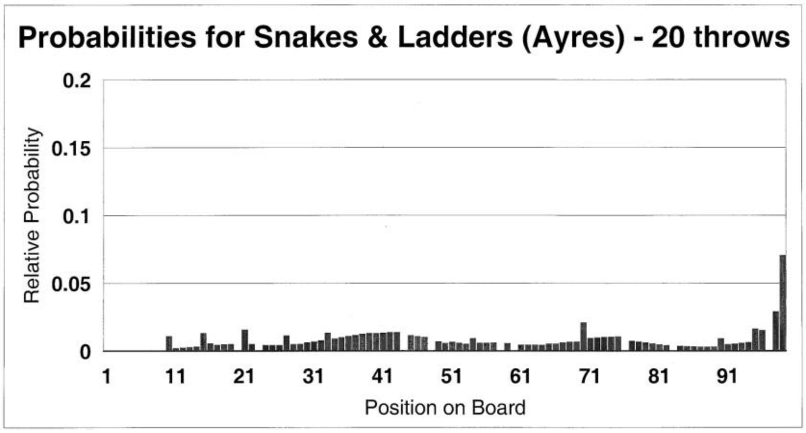


Figure 2 – Markov Rounds to Win (compare figure 3)

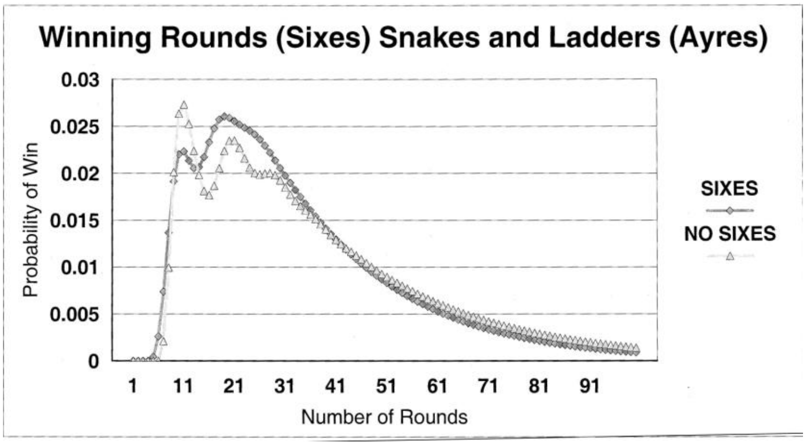


Figure 3 – Monte Carlo Rounds to Win (compare figure 2)

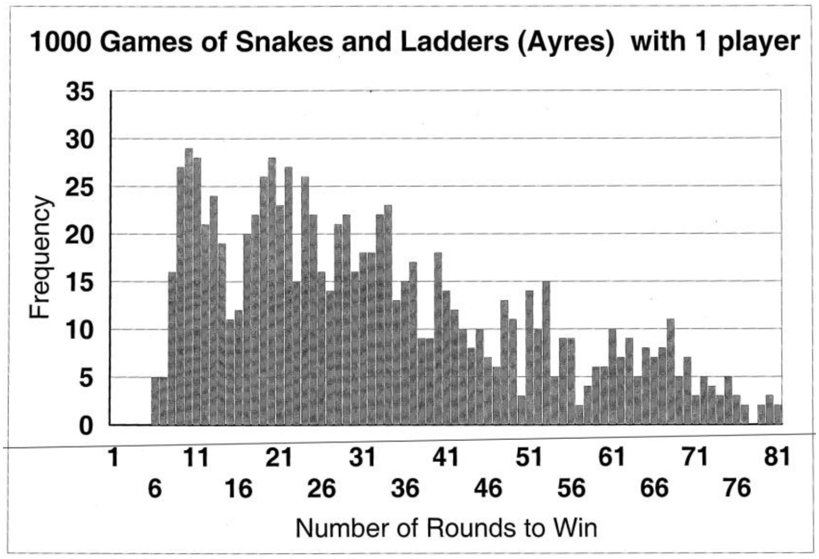


Figure 4 – 'Snapshots' of the Board (compare figure 1)

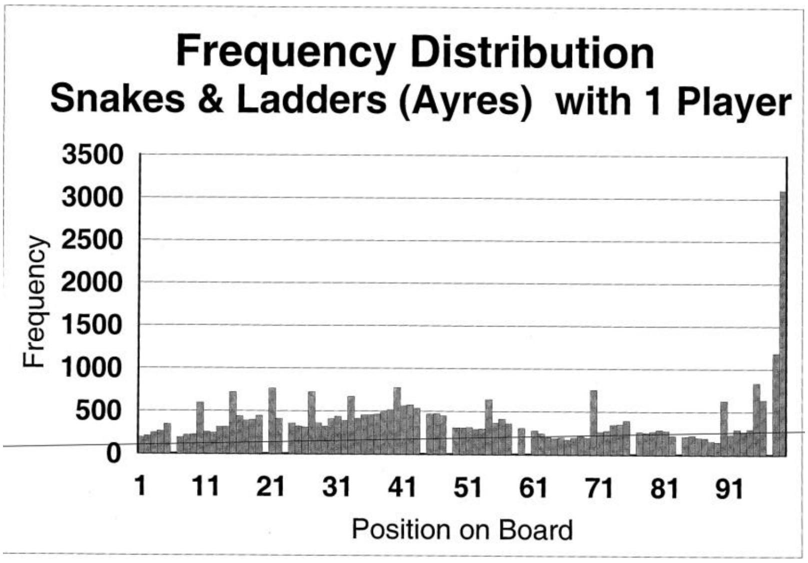
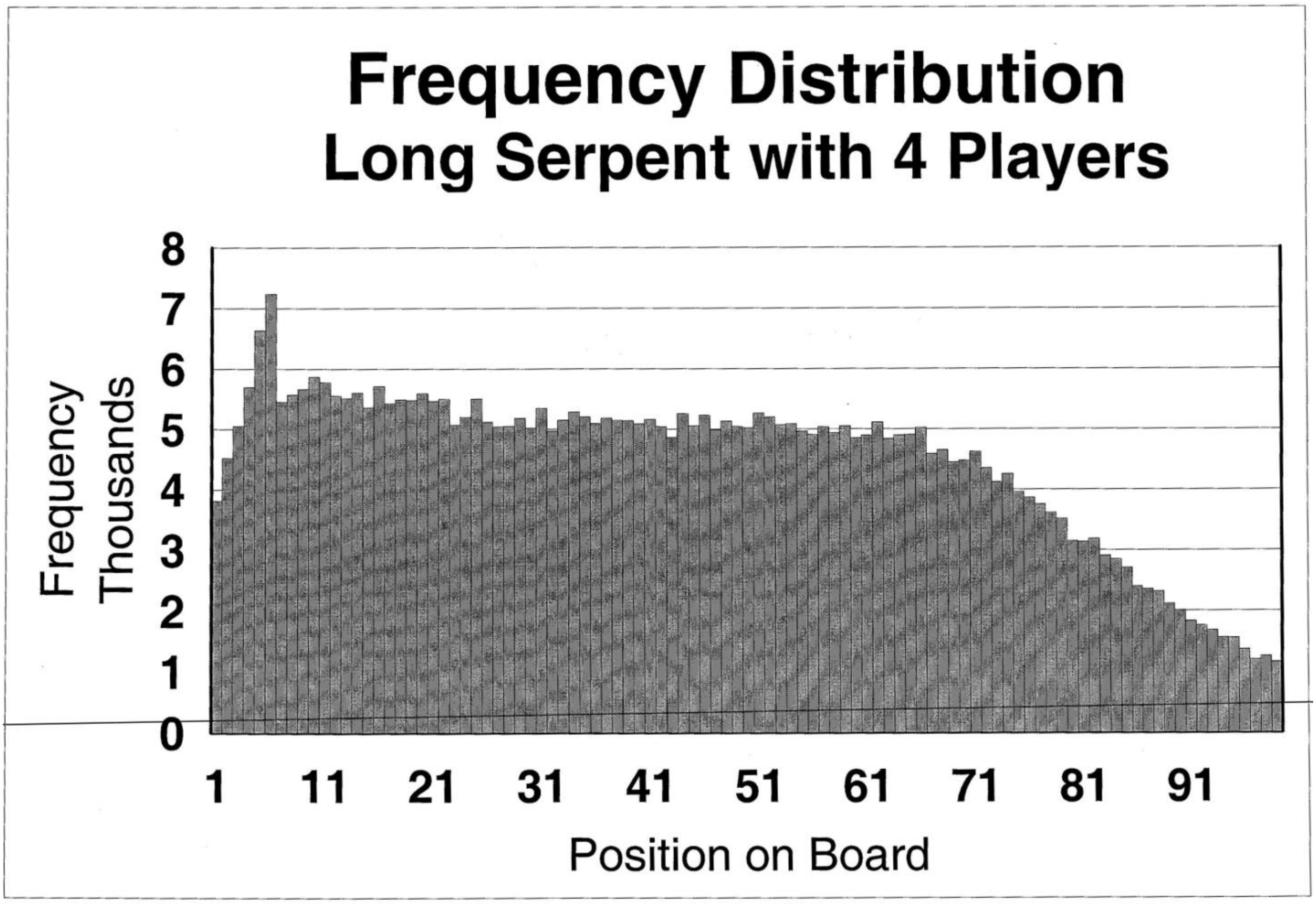


Figure 5 – the Approach to Snakes and Ladders: (a) Long Serpent



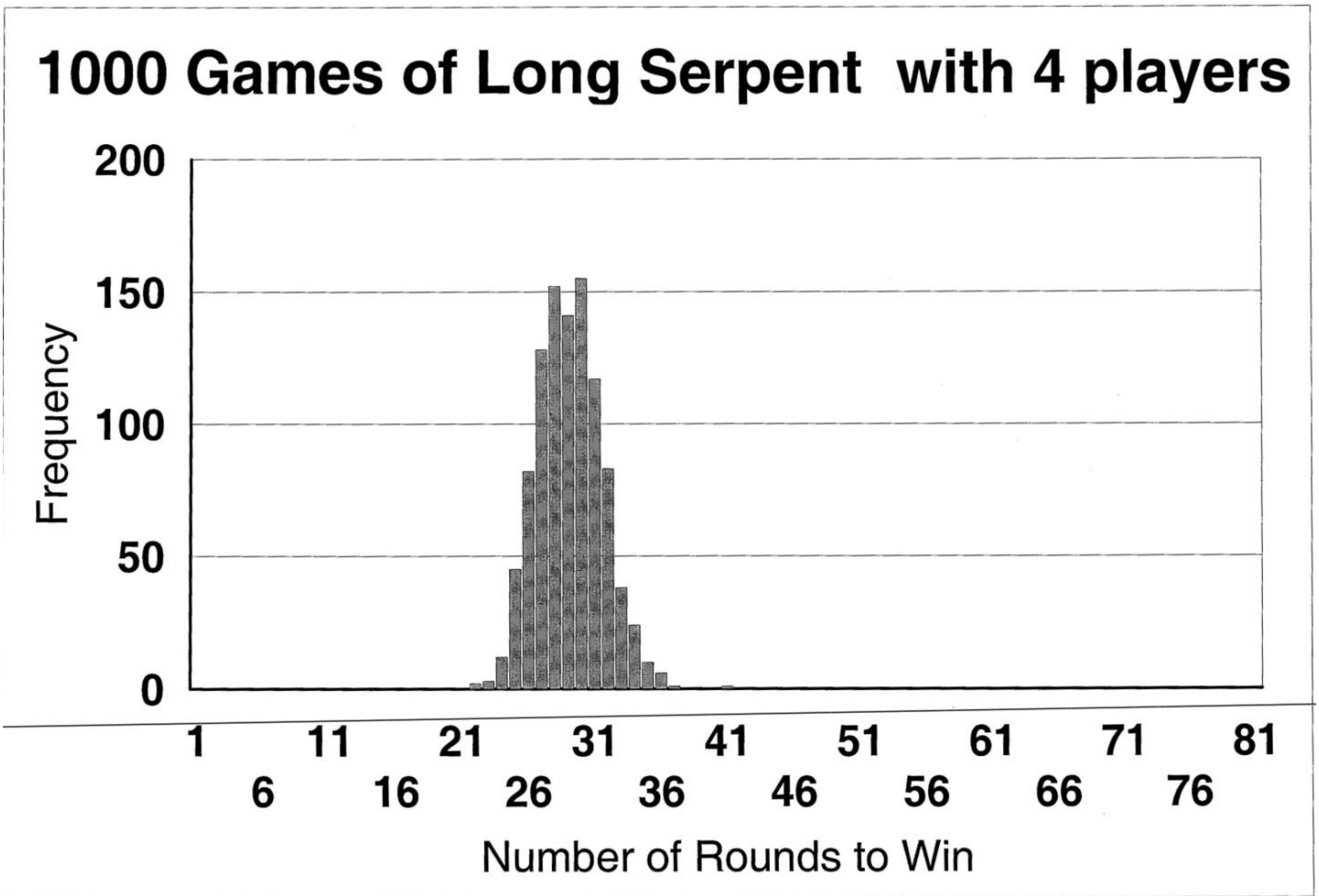
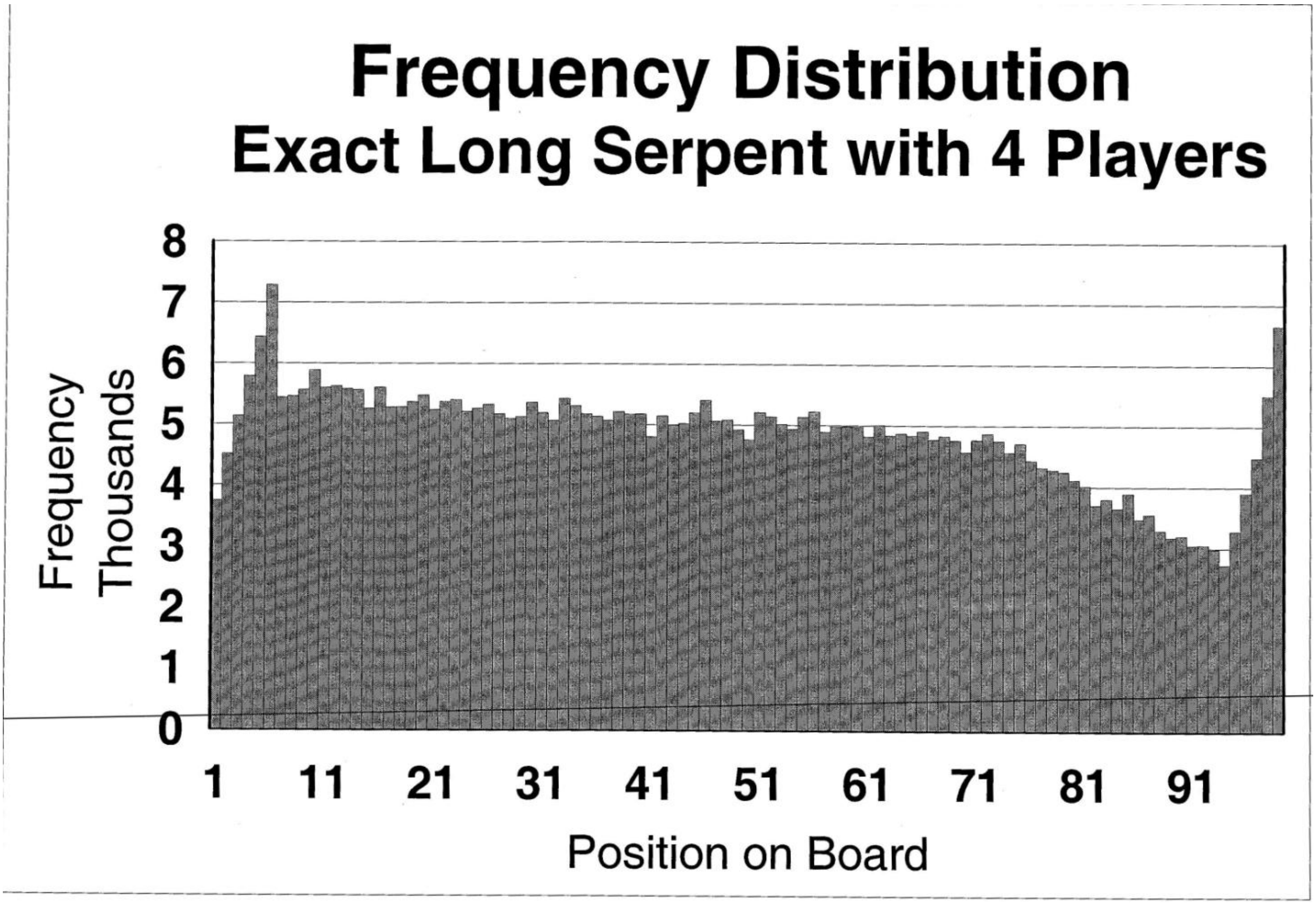


Figure 5 – the Approach to Snakes and Ladders: (b) Exact Long Serpent



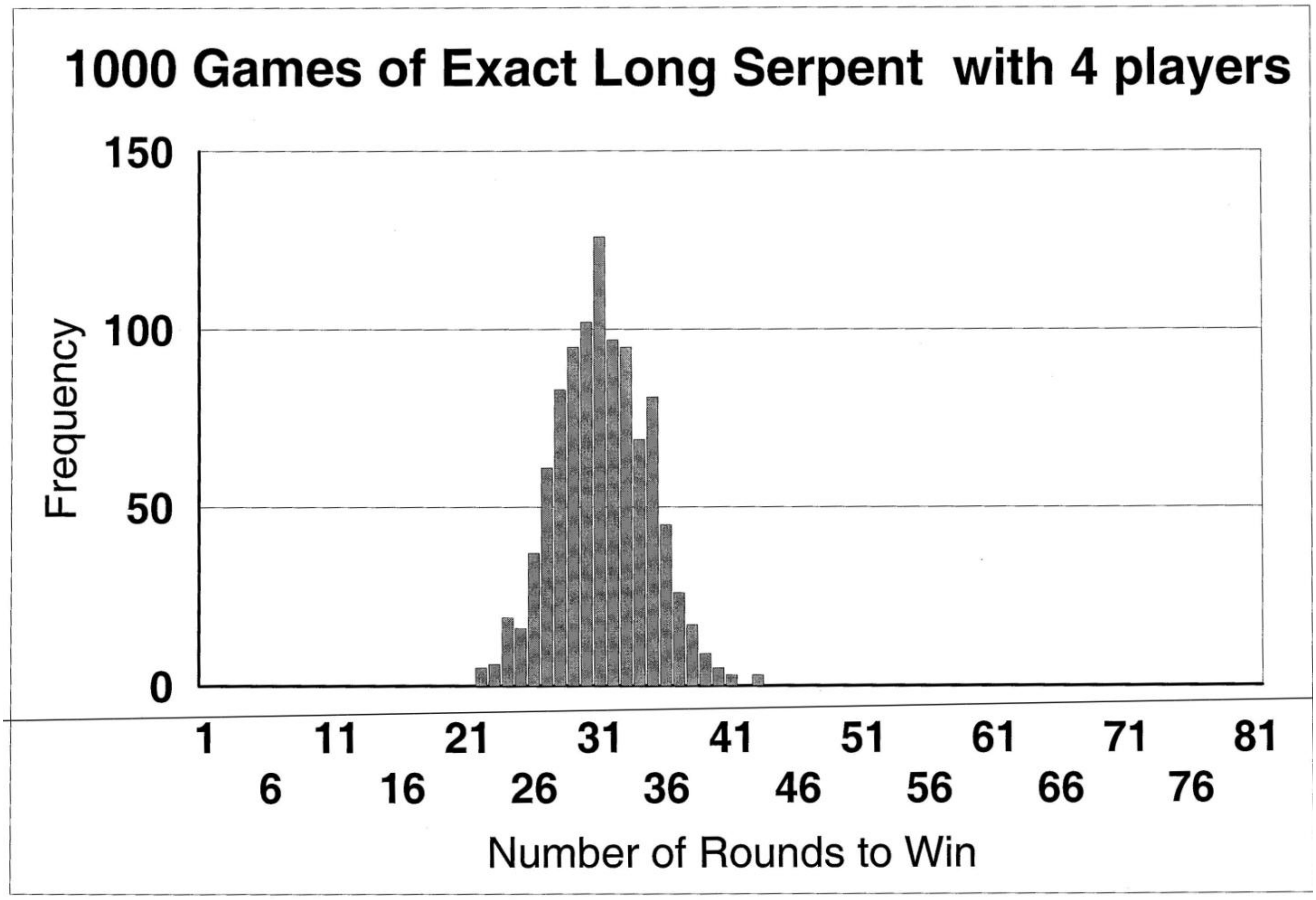
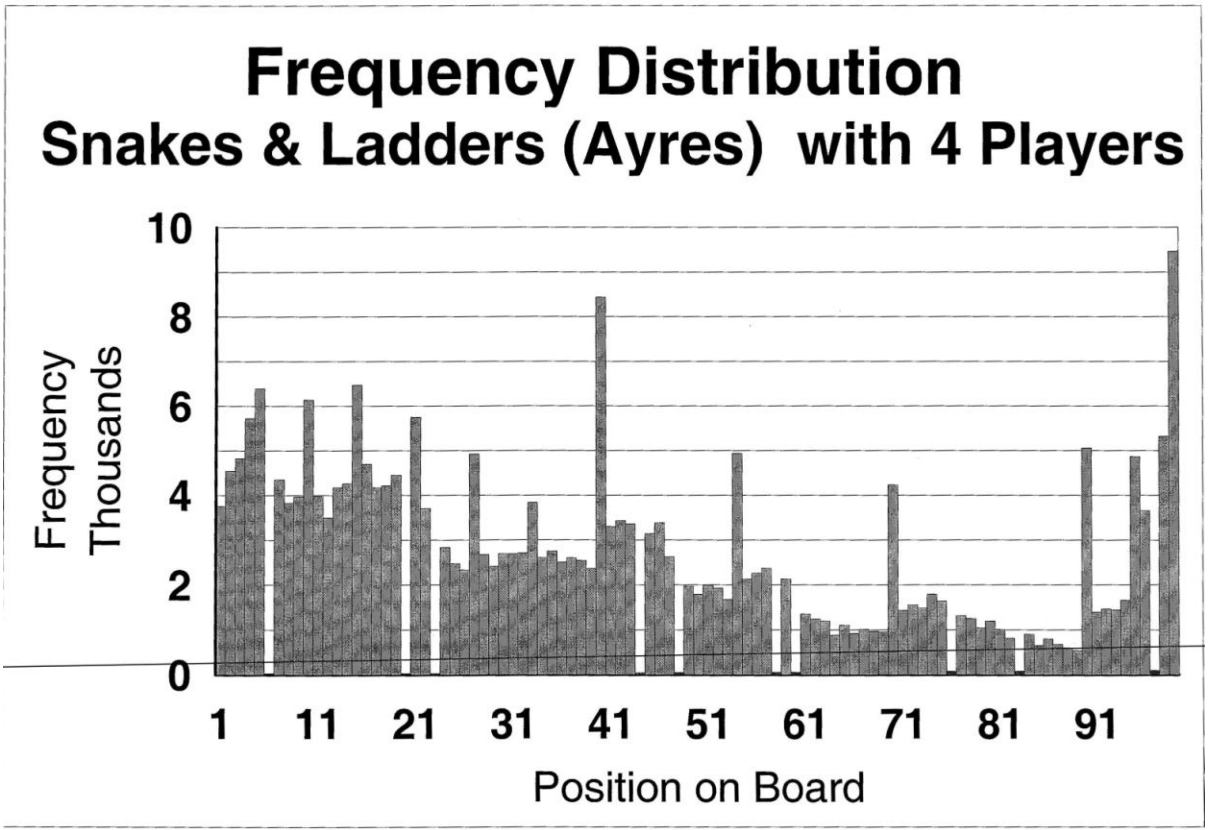


Figure 5 – the Approach to Snakes and Ladders: (c) Snakes and Ladders



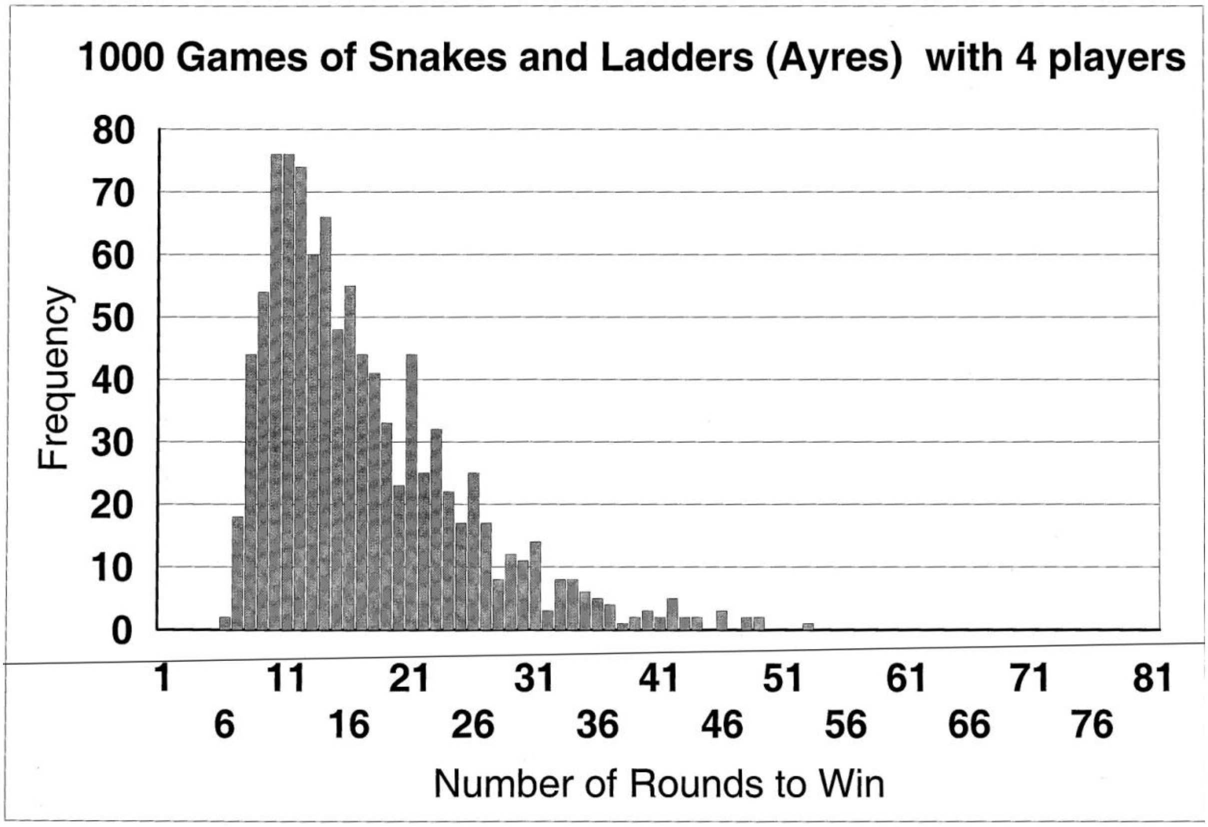
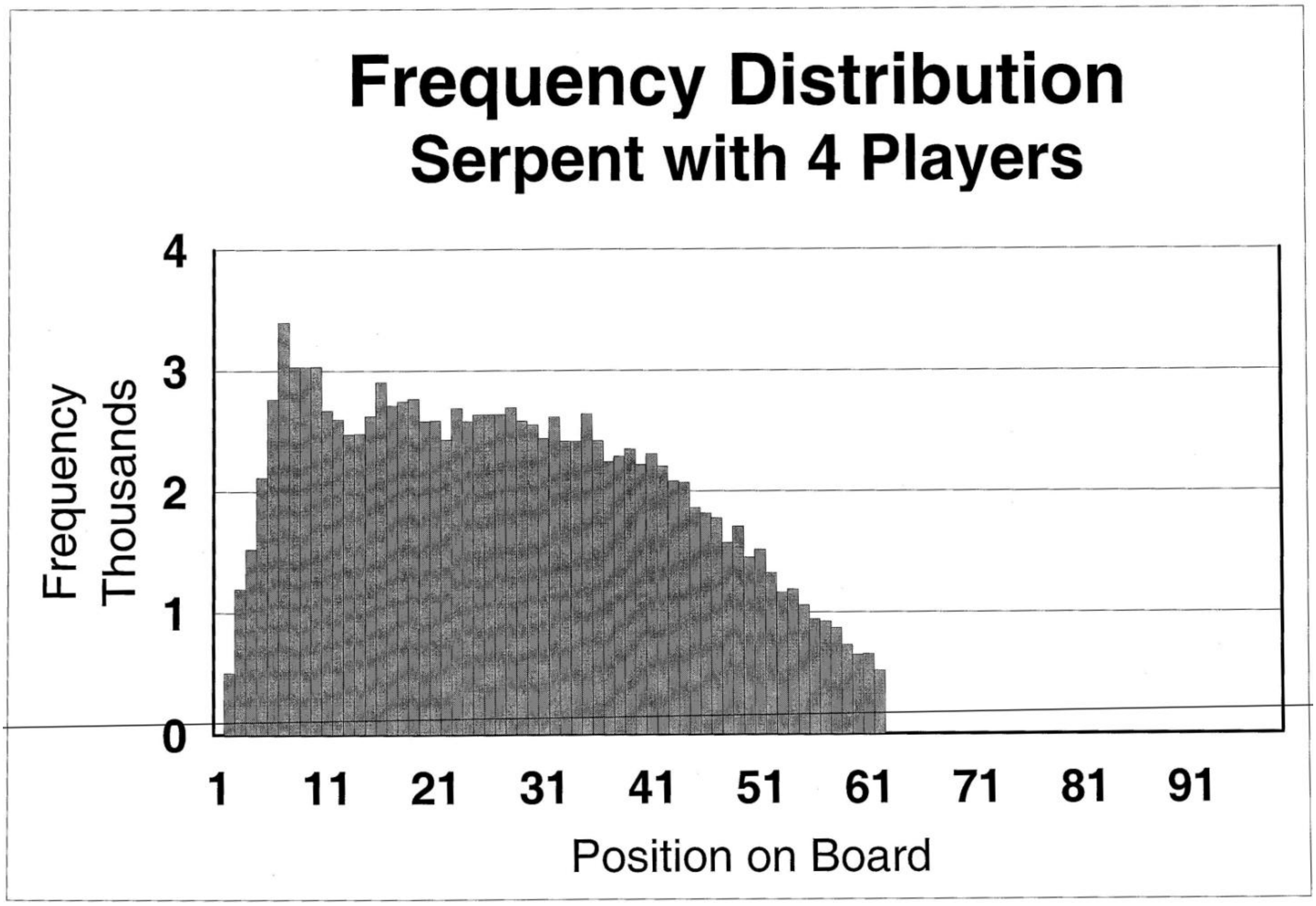


Figure 6 – the Approach to Goose: (a) Serpent



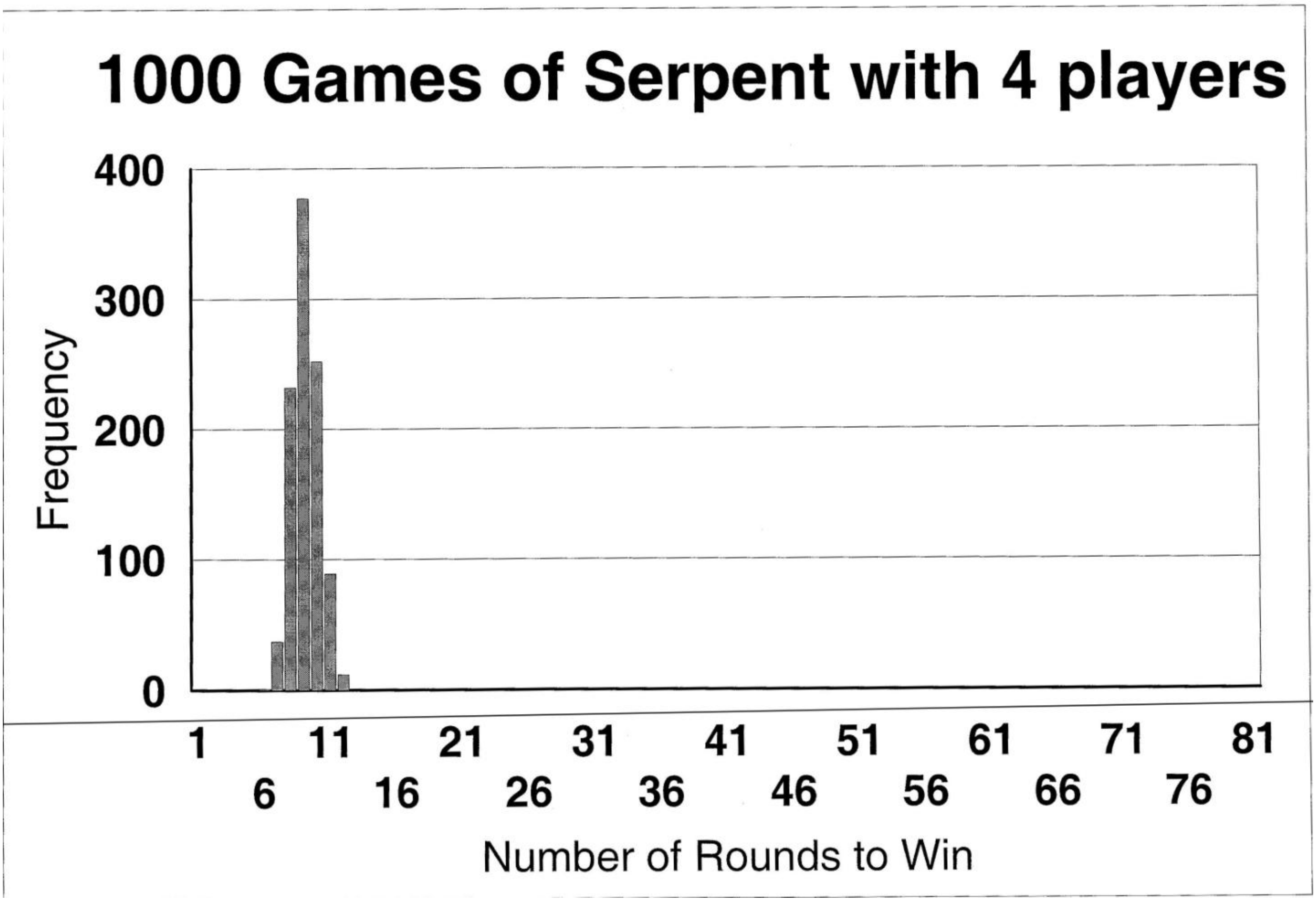
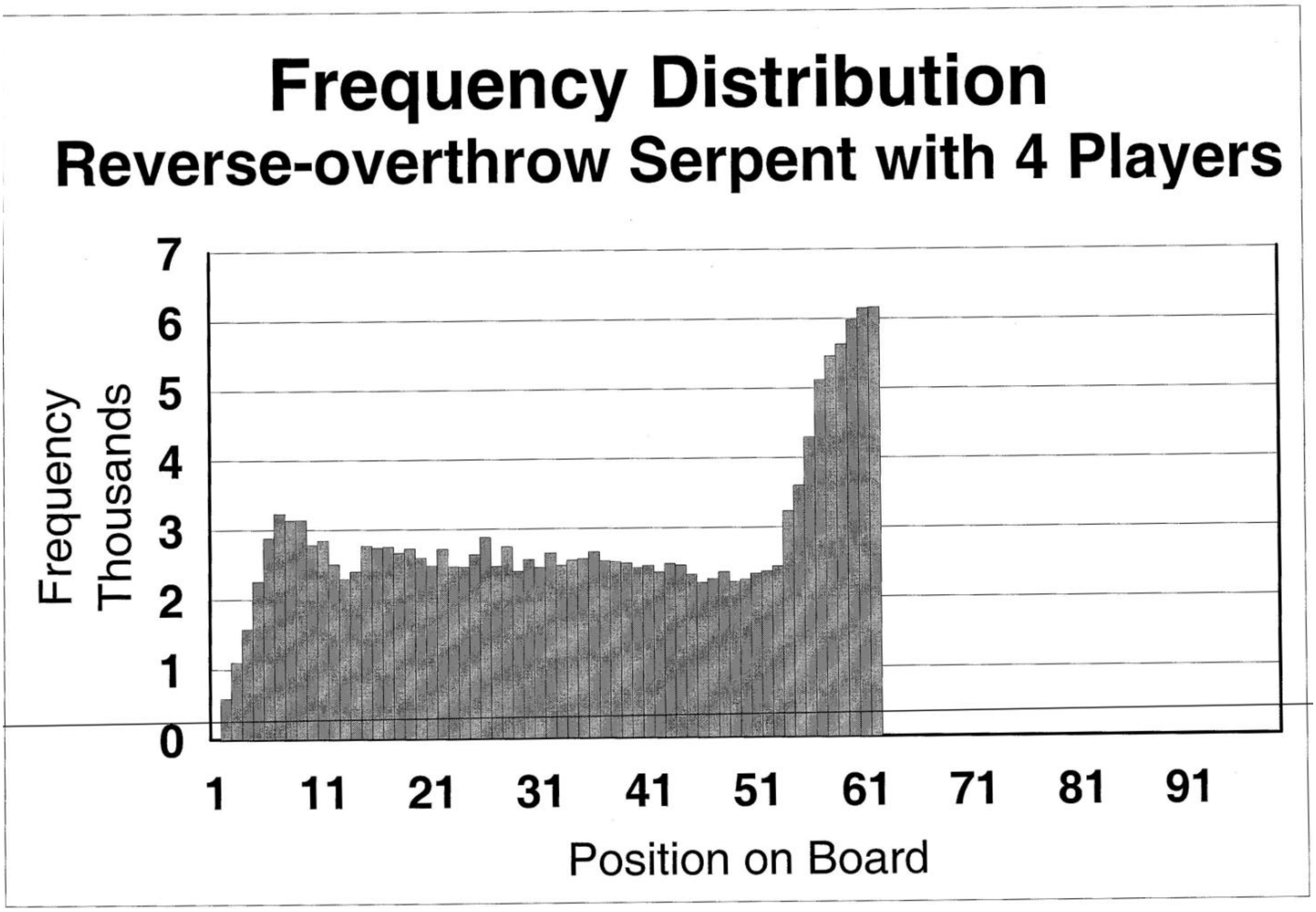


Figure 6 – the Approach to Goose: (a) Reverse-Overthrow Serpent



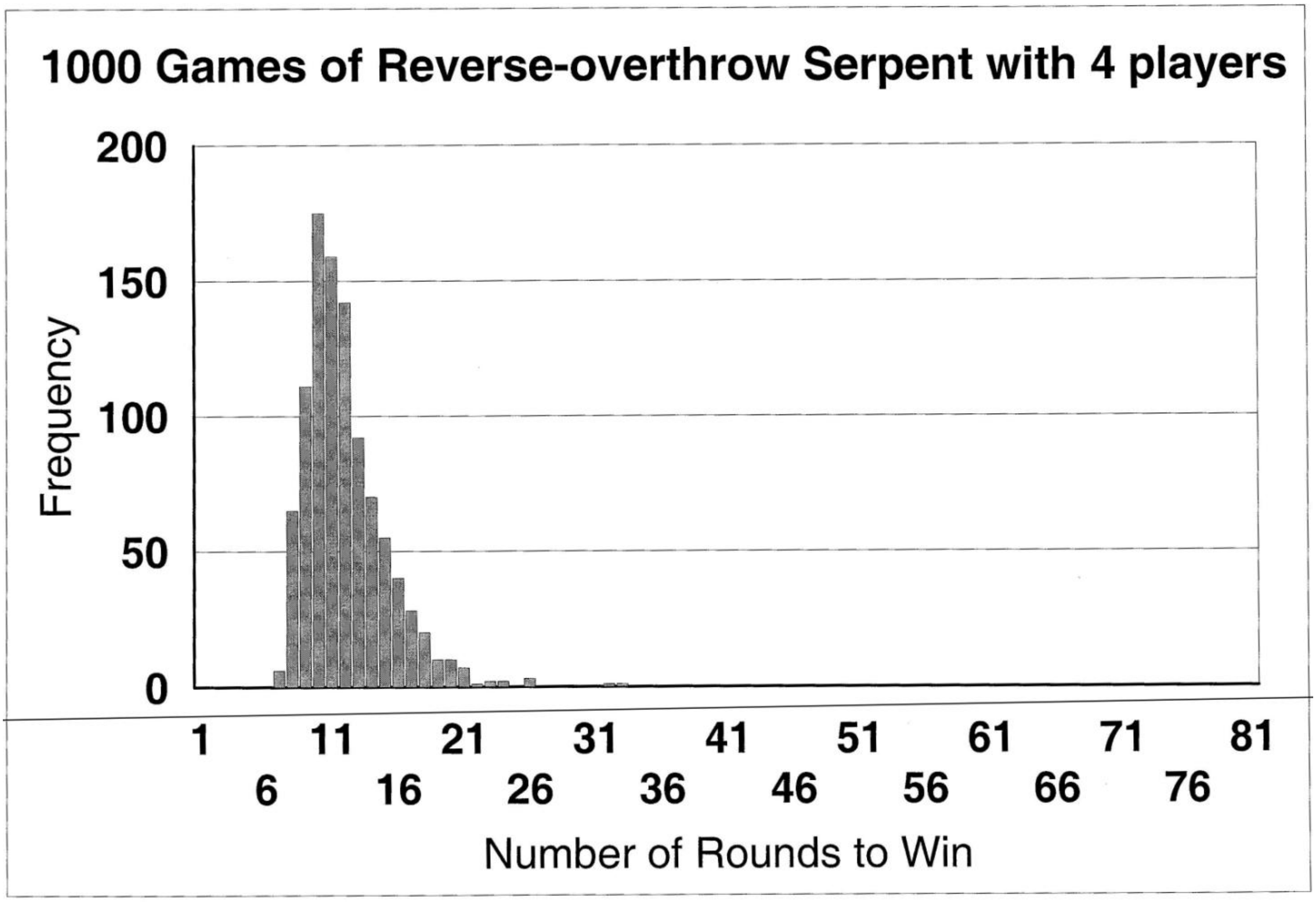
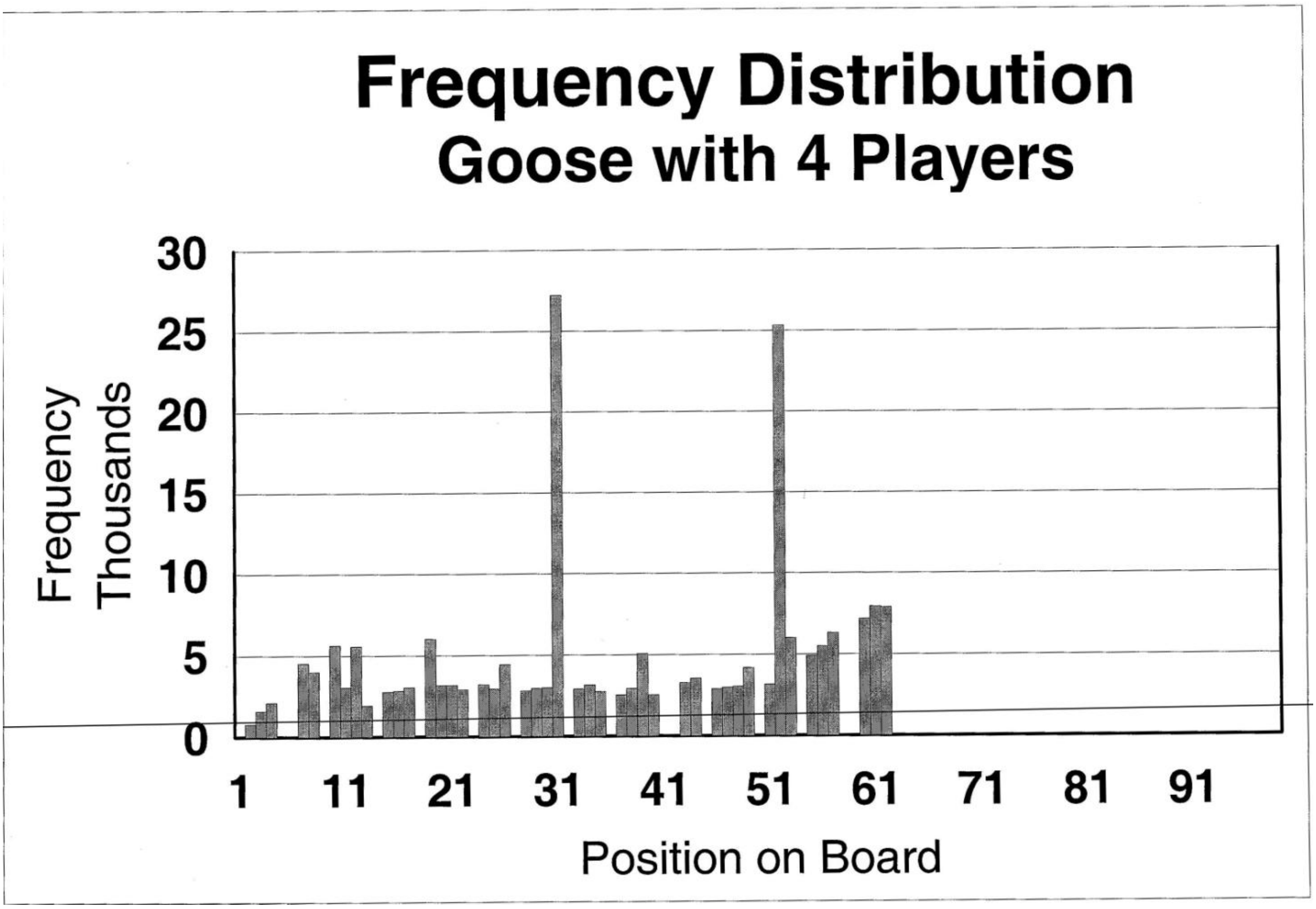
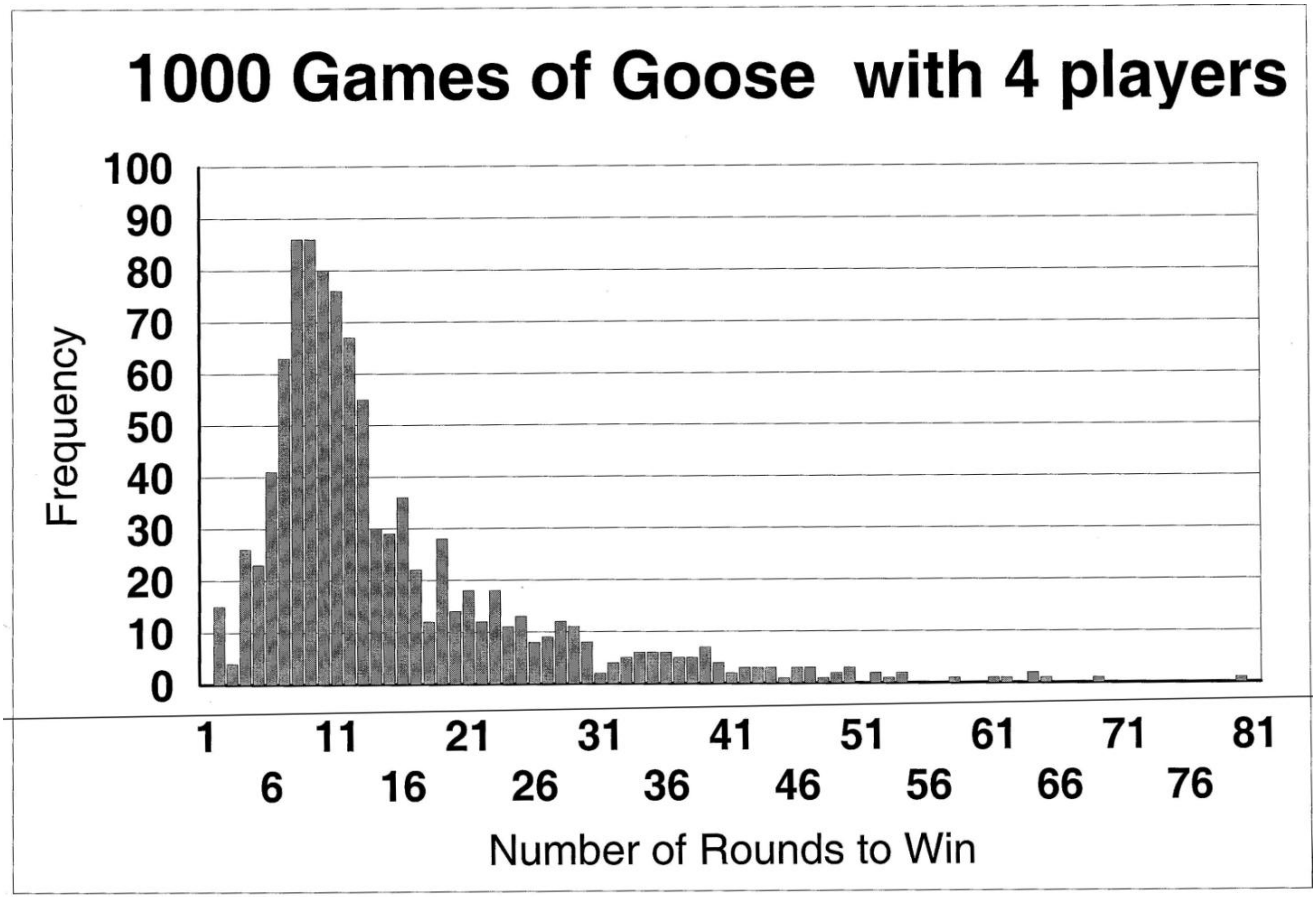


Figure 6 – the Approach to Goose: (c) Goose





AVERAGE NUMBER OF ROUNDS TO WIN

Number of players	1	3	4	6	8
APPROACH TO SNAKES AND LADDERS - Single dice					
LONG SERPENT	29.1	28.9	28.9	28.7	
EXACT LONG SERPENT	33.5	31.5	31.2	31.1	
S & L (AYRES)	37.0	19.4	17.2	15.1	
APPROACH TO GOOSE - double dice					
SERPENT 2dice	9.5	9.2	9.2	9.1	9.1
REVERSE SERPENT	21.8	12.8	12.04	11.55	11.47
GOOSE		31.2	15.1	11.2	10.4

STANDARD DEVIATION OF NUMBER OF ROUNDS

Number of players	1	3	4	6	8
APPROACH TO SNAKES AND LADDERS - Single dice					
LONG SERPENT	2.5	2.8	2.8	2.7	
EXACT LONG SERPENT	6.0	3.5	3.5	3.6	
S & L (AYRES)	25.2	10.5	8.0	6.1	
APPROACH TO GOOSE - double dice					
SERPENT 2dice	1.1	1.1	1.0	1.0	1.0
REVERSE SERPENT	14.1	4.0	3.1	2.4	2.2
GOOSE		19.5	10.7	5.7	5.4

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